

NAG-5-2210

GRANT  
IN-61-CR

## Stability by Linear Processes

Problem: To find a quick way to determine if the origin  $O$  in the  $CIT-0$  complex plane  $C$  within the image of  $Q$  under the multilinear function  $p(j\omega, q) = f(q) + g(q)j$  for fixed  $\omega$ .

Notation: Parameter Space  $R^n$

 $C$ 

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1.  $A_1, A_2, \dots$  in  $Q \subset R^n$   
are preimages of  
 $A_1', A_2', \dots$   
under  $p(j\omega, q)$

$A_1', A_2', \dots$   
points in  $C$

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2.  $l_1$  edge and  
 $l_2, \dots, l_n$   
line segments where  
 $l_1 \perp l_2 \dots \perp l_n$

$l_1', l_2', \dots, l_n'$   
line segments in  $C$   
which are images of line  
segments  $l_1, l_2, \dots, l_n$   
respectively from  $Q$ .

Without loss of generality, assume that the point  $(q_1^-, q_2^-, \dots, q_n^-)$  and the point  $(q_1^+, q_2^-, \dots, q_n^-)$  are the endpoints of the edge  $l_1$  and the point  $A_1 \in l_1$  would be  $(q_1, q_2^-, \dots, q_n^-)$  then the point  $(q_1, q_2^-, \dots, q_n^-)$  and the point  $(q_1, q_2^+, q_3^-, \dots, q_n^-)$  are the endpoint of  $l_2$  and the point  $A_2 \in l_2$  is  $(q_1, q_2, q_3^-, \dots, q_n^-)$ . So the point  $(q_1, \dots, q_i^-, q_{i+1}^-, \dots, q_n^-)$  and the point  $(q_1, \dots, q_i^+, q_{i+1}^-, \dots, q_n^-)$  are the endpoints of  $l_i$  and the point  $A_i \in l_i$  is  $(q_1, \dots, q_i, q_{i+1}^-, \dots, q_n^-)$  and so forth.

## Algorithm:

1. Map any edge,  $l_1$ , of  $Q \subset R^n$  to the line segment  $l_1'$  in  $C$ .
2. If the line through  $O$  and  $A_1'$  is not perpendicular to  $l_1'$  then we are finished.
3. Determine the point  $A_1'$  on  $l_1'$  that is closest to  $O$ , if the line through  $O$  and  $A_1'$  is perpendicular to  $l_1$ .
  - a) Once  $(q_1^-, q_2^-, \dots, q_n^-)$  and  $(q_1^+, q_2^-, \dots, q_n^-)$  is mapped to  $C$ , say  $f(q_1^\pm) = f(q_1^\pm, q_2^-, \dots, q_n^-)$  and  $g(q_1^\pm) = g(q_1^\pm, q_2^-, \dots, q_n^-)$
  - b) Then the slope

$$m = \frac{g(q_1^-) - g(q_1^+)}{f(q_1^-) - f(q_1^+)}$$

is calculated.

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c) The closest point  $A_1'$  ( $x, y$ ) to 0 in  $C$  is

$$x = \frac{mf(q_1) - g(q_1)}{(m + 1/m)}$$

$$y = -x/m$$

d) Consider the preimage  $A_1$  of  $A_1'$ . It is easy to determine the point  $A_1$  lies on the edge  $l_1$  on  $Q$  by linearity.

4. Construct the line  $l_2 \perp l_1$  through  $A_1$  in  $Q$  along with  $i$ th axis.

a) Since all of the  $q$ 's are fixed except  $q_i$  then the  $i$ th coordinate of  $A_1$  is founded by

$$q_i = \frac{x - \text{all linear factors not containing } q_i}{\text{all linear factor containing } q_i}$$

5. Repeat the process for the lines  $l_i$ ,  $i = 3, \dots, n$  as was done for  $l_2$ .

The speed of the algorithm can be determined by examining the best/worst case scenario. The best case is that the first edge tried does not have a perpendicular through 0 which means we exit the algorithm. The worst case is that all of the  $n-1$  directional edges have perpendiculars. This requires  $n$  iterations. Therefore the average case requires  $(n+1)/2$  iterations. One iteration includes the following calculations:

1.  $f$  and  $g$  for an edge
2. the slope of the line segment in  $C$
3. the point  $(x, y)$  which is perpendicular to the line through 0
4. determining if  $(x, y)$  lies within the line segment
5. mapping  $(x, y)$  to the preimage

Conjecture 1:  $p(j\omega, q) \in \text{Hurwitz} \forall q \in Q$  Iff for some  $i \in \{1, \dots, n\}$  the line through 0 is not perpendicular to  $l_i'$

Conjecture 2:  $p(j\omega, q) \notin \text{Hurwitz} \forall q \in Q$  Iff  $d_i = \|A_i' - 0\|$ ,  $i = 1, \dots, n$  then  $d_i > d_{i+1}$  for each  $i = 1, \dots, n-1$ .

To prove the above conjectures we are considering the following direction.

Lemma 1:

If for all edges of  $Q$  map to line segment in  $C$  no perpendicular to these segments pass through 0, then all line segments  $l_i'$ ,  $i=2, \dots, n$  do not have a perpendicular through 0.

Lemma 2:

Given any line segment  $l_j$  in  $Q$  parallel  $j$ th axis consider all line segments  $l_{j+1}^k$ ,  $k=1, 2, \dots, n-1$  such that  $l_{j+1}^k$  along with  $(j+1)$  axis then  $l_{j+1}^k$  are all on the same side of  $l_j'$ .

Lemma 1 has already been proven.

Example:

$$f(q_1, q_2, q_3, q_4) = q_1 q_2 q_3 q_4 + q_1 + q_3 + q_2 q_3 + 1$$
$$g(q_1, q_2, q_3, q_4) = q_1 q_2 q_3 + 2 q_1 + q_2 + q_3 + q_4$$

1st pass

endpoints  $(-2, -2, -2, -2)$   $(-2, -2, -2, 2)$   
image in  $\mathbb{C}$   $(17, -18)$   $(-15, -14)$   
 $\perp$  point  $(-1.953846, -15.63077)$  inside line segment

2nd pass

$(-2, -2, -2, .3692307)$   $(2, -2, -2, .3692307)$   
 $(-1.953846, -15.63077)$   $(7.953846, 8.36923)$   
 $\perp$  point  $(3.843794, -1.586797)$  inside line segment

3rd pass

$(.340662, -2, -2, .3692307)$   $(.340662, -2, 2, .3692307)$   
 $(3.843794, -1.586797)$   $(-1.16247, -.3120933)$   
 $\perp$  point inside  $(-.1454044, -.5710602)$  inside line segment

4th pass

$(.340662, -2, 1.187366, .3692307)$   $(.340662, 2, 1.187366, .3692307)$   
 $(-.1454044, -.5710603)$   $(5.20146, 5.046901)$   
 $\perp$  point inside  $(.2088863, -.1988064)$  inside line segment

Then  $0 \in \text{Im}\{p(s, Q)\}$

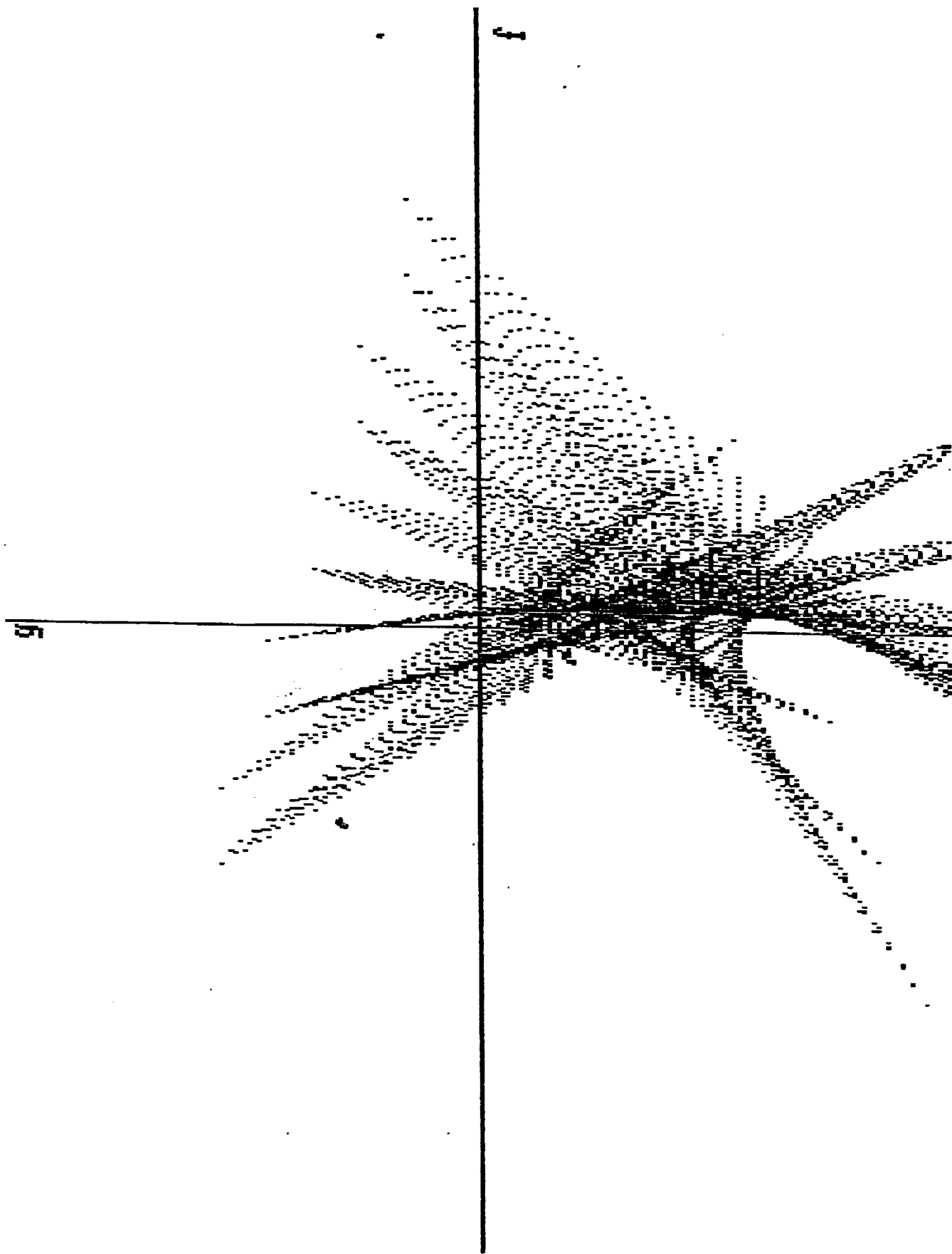


Figure 1a

2

4

2

 $\ell_2^1$  $q^+$ 

A2

 $l'_3$ 

A'3

A'

Figure 2a

$$f(q_1, q_2, q_3, q_4) = q_1 q_2 q_3 q_4 + q_1 + q_2 + q_3 + q_2 q_3 - 2 q_1 q_2 q_3 - 10$$

$$g(q_1, q_2, q_3, q_4) = -q_1 q_2 q_3 + 2 q_1 + q_2 + q_3 + q_4 - q_2 q_3 - q_3 q_4 + 2$$

1st pass

endpoints  $(-2, -2, -2, -2)$   $(-2, -2, -2, 2)$

image in  $\mathbb{C}$   $(20, -8)$   $(-12, 4)$

$\perp$  point  $(-.1643836, -.4383562)$  inside line segment

2nd pass

$(-2, -2, -2, .5205479)$   $(2, -2, -2, .5205479)$

$(-.1643834, -.4383562)$   $(-19.83562, -8.438356)$

$\perp$  point  $(.1296431, -.31878)$  outside line segment

Then  $0 \notin \text{Im}\{p(s, Q)\}$

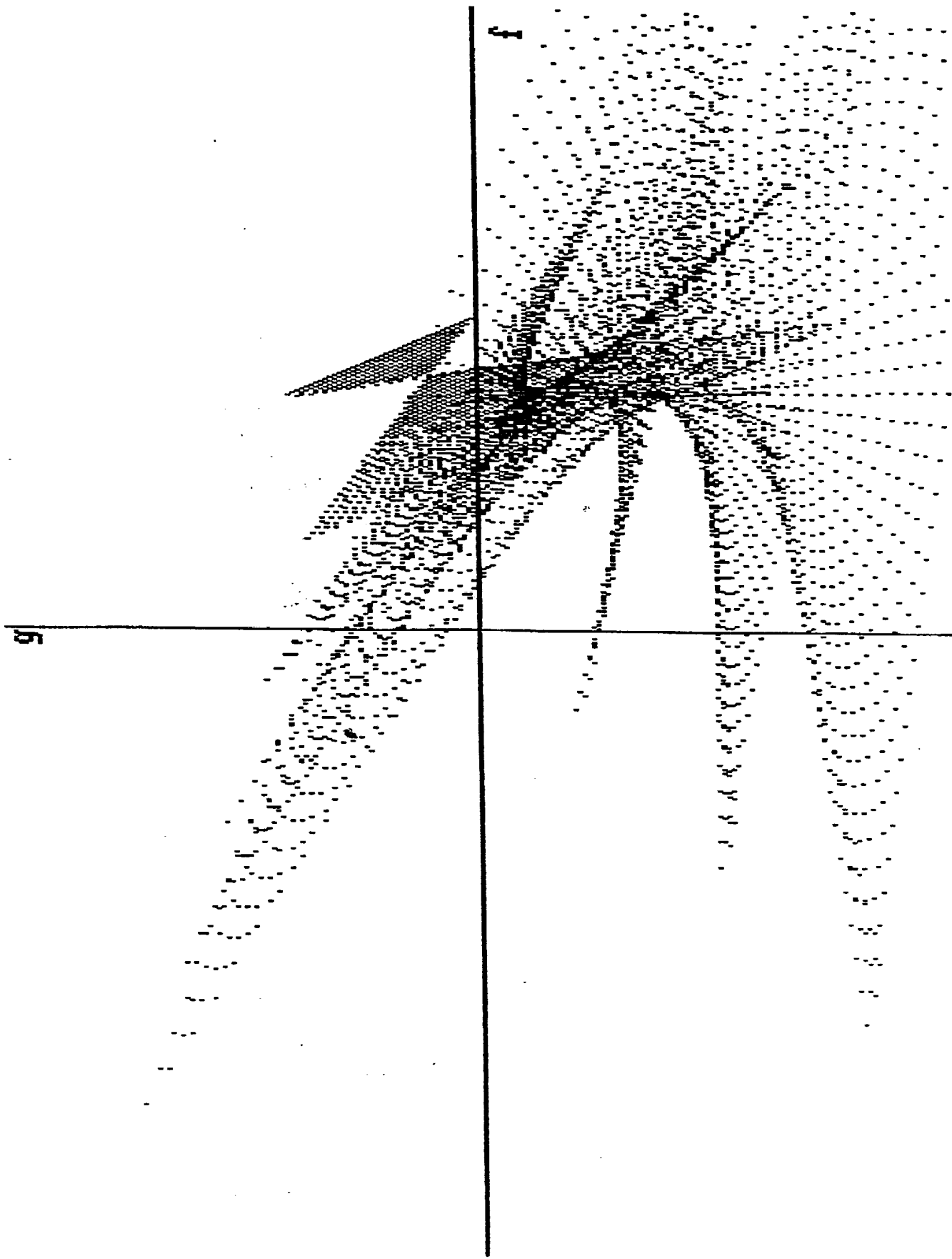


Figure 1b

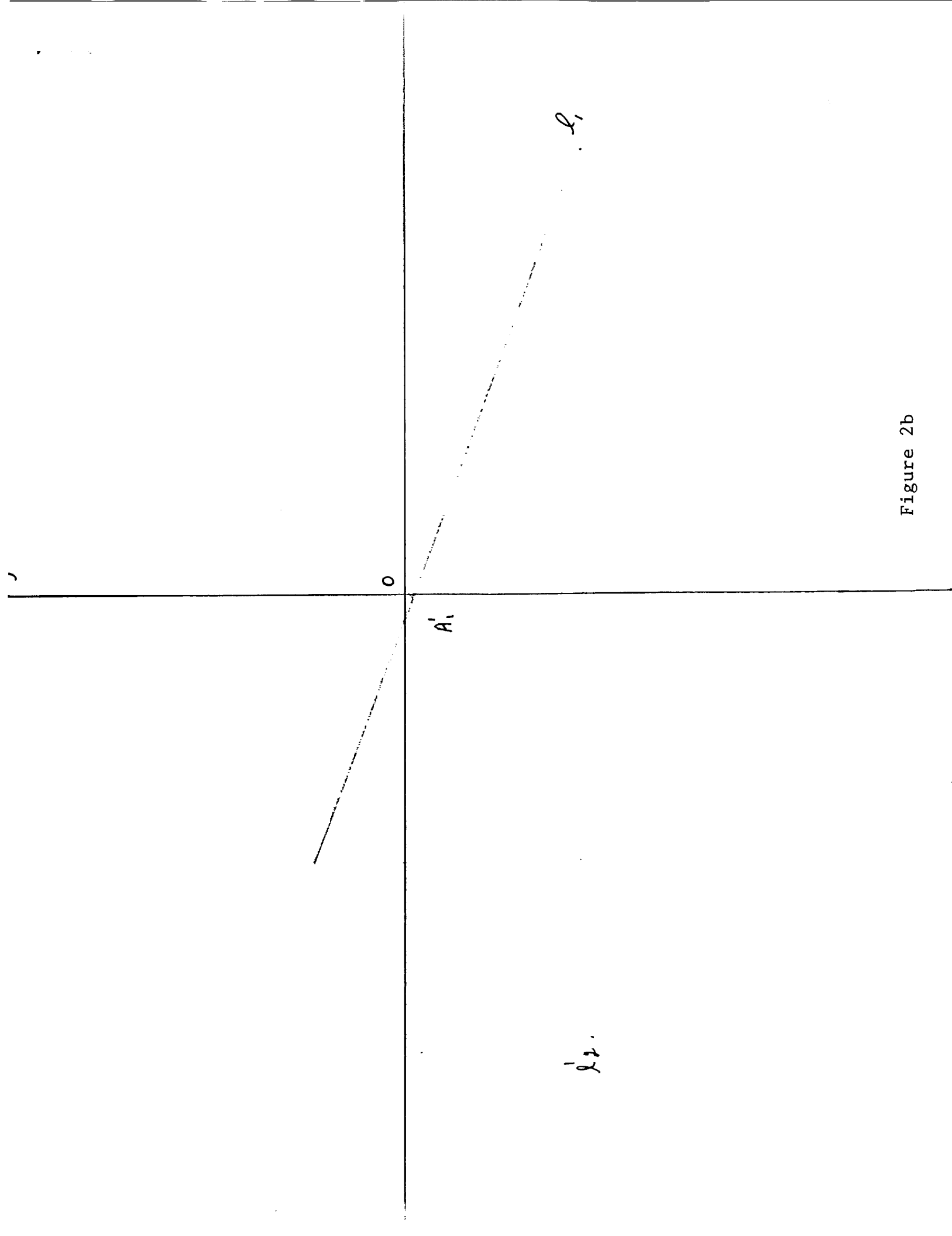


Figure 2b